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The theory of heat and moisture transfer in porous media revisited†

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Abstract—In this paper a review is presented of the present status of the theory of combined heat and moisture transfer in porous media, developed by J. R. Philip and the author in the mid-1950s. First, attention is drawn to the limitations of the theory and the assumptions underlying it. Next, attempts to test the theory by laboratory and field experiments are briefly discussed, leading to the conclusion that the usefulness of the theory in describing and analysing the experiments was proven, but that doubts remain about its predictive value. These doubts are a consequence of: (a) the limitations of the theory; (b) uncertainty about the quality of the experimental procedures and data. Remarks are made on hysteresis and its possible influence. It is concluded that experiments aimed at a study of the behaviour of non-isothermal systems subjected to hysteresis are needed. Finally, the problem of the definition and use of an apparent thermal conductivity is analysed. In the original papers two alternatives were presented. An expression for the phase average of the vapour flux density is derived. A numerical example is presented and suggestions are made concerning the proper choice between the alternatives.

1. INTRODUCTION

THE CONTINUING interest in simultaneous heat and mass transfer in porous media, exemplified by the organization of this colloquium, is based on the following two facts:

(a) the importance of the subject for a—still expanding—multitude of applications;

(b) the lack of a comprehensive and satisfactory theory of the subject, despite much important work that has been done.

Considering the wide range of porous media and their application, met in practice, it is even doubtful whether a universal theory can be developed.

The work on combined heat and moisture transfer in soils, which Dr J. R. Philip and I [1, 2] did more than 25 years ago, has become generally known and applied, also to porous media other than soils. I believe that this is due to the fact that we gave theoretical expressions for the macroscopic diffusivities, which occur in the equations describing combined heat and mass transfer, that make it possible to calculate them from only two functionals. These are the dependence of moisture potential (ψ) and hydraulic conductivity (K) of the porous medium on its volumetric moisture content (θ_l).§ In addition, the thermal conductivity (λ) of the porous medium must be known, also as a function of its moisture content. It should be noted that the quantities mentioned depend on temperature (T) as well. The dependent variables in the theoretical treatment are θ_l and T.

After not having been actively engaged in the field for a considerable time, I had various opportunities to take part in discussions on the subject and to become interested in it again after my retirement from the Eindhoven University of Technology $2\frac{1}{2}$ years ago. There is a French saying: "On revient toujours à ses premières amours", which is applicable in my case. In this connection I express my sincere thanks to the Scientific and Local Committee and, in particular, to its Chairman, Dr M. G. Martin, for the invitation extended to me to participate and to present this lecture.

In this paper I shall attempt to evaluate the present situation with respect to the theory developed by Philip and me (for brevity further referred to as 'the theory'). I shall do so by first listing the principal limitations of the theory and the assumptions made in its derivation. Next I shall say something about attempts made to test the theory experimentally. Finally, I shall present some ideas on the concept of an apparent thermal conductivity of a moist porous medium.

Space does not permit me to present the theory and I presume that it will be known to most. A summary of the principal equations is given in the Appendix.

Also I do not pretend to give a comprehensive

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§ The terminology refers to porous media containing water, but the theory could also be applied to media containing another liquid. In this paper I shall mostly speak of water and moisture.

NOMENCLATURE

a	volumetric air content [—]
c, c_p	specific heat [$\mathrm{J\,kg^{-1}\,K^{-1}}$]
C	volumetric heat capacity [$\mathrm{J\,m^{-3}\,K^{-1}}$]
D	diffusion coefficient of water vapour in air [$\mathrm{m^2\,s^{-1}}$]
D_T	macroscopic diffusivity for moisture transport due to ∇T [$\mathrm{m^2\,s^{-1}\,K^{-1}}$]
D_θ	macroscopic diffusivity for moisture transport due to $\nabla\theta_l$ [$\mathrm{m^2\,s^{-1}}$]
e	empirical factor [—]
g	acceleration due to gravity [$\mathrm{m\,s^{-2}}$]
h	relative humidity [—]
H	specific enthalpy [$\mathrm{J\,kg^{-1}}$]
$\mathbf{k}$	unit vector in the z-direction [—]
K	hydraulic conductivity [$\mathrm{m\,s^{-1}}$]
L	latent heat of evaporation [$\mathrm{J\,kg^{-1}}$]
M	molar mass [$\mathrm{kg\,mol^{-1}}$]
p_v	partial pressure of water vapour [Pa]
P	total gas pressure [Pa]
$\mathbf{q}$	flux density for mass [$\mathrm{kg\,m^{-2}\,s^{-1}}$]
$\mathbf{q}_h$	flux density for heat [$\mathrm{J\,m^{-2}\,s^{-1}}$]
R	universal gas constant [$\mathrm{J\,mol^{-1}\,K^{-1}}$]
S	porosity [—]
t	time [s]
T	temperature [K]
x, y	horizontal coordinate [m]
z	vertical coordinate, positive downward [m].

Greek symbols

ε	low value of θ_l [—]
ζ	$(\nabla T)_a/\nabla T$ [—]
θ	volumetric moisture content [—]
λ	(apparent) thermal conductivity [$\mathrm{W\,m^{-1}\,s^{-1}}$]
ρ	density [$\mathrm{kg\,m^{-3}}$]
ψ	moisture potential [m].

Subscripts

0	reference value
a	air
d	drying
h	heat
k	critical
l	liquid
m	moisture
M	matrix
r	reversal
s	saturation
v	vapour
w	wetting.

Superscripts

$'$	pure conduction
$*$	macroscopic inclusion of vapour transfer
$+$	microscopic inclusion of vapour transfer
μ	microscopic quantity.

review. I know that much relevant work of the past decades is not mentioned in this paper.

2. LIMITATIONS AND ASSUMPTIONS

The principal inherent limitations of the theory are:

(1) Hysteresis in the relation between the moisture potential and the moisture content is not taken into account.

(2) The theory does not apply when the matrix is not rigid.

(3) The porous medium must be homogeneous and isotropic in a macroscopic sense.

(4) Phenomena of boiling and freezing and thawing are not included.

(5) Surface phenomena at the interface between the matrix and the liquid are not taken into account, nor are Knudsen effects in the gas phase.

Some comments on these limitations are given below:

(Ad1) Hysteresis will be discussed in Section 4.

(Ad2) The limitation implies that the theory does not apply to shrinking and swelling materials, such as clay soils, or to cases where the matrix is deformed as a consequence of internal or external forces. I shall not discuss these processes. Much work on shrinking and swelling soils has been performed at the CSIRO Division of Environmental Mechanics by Philip and co-workers. The reader is referred to refs. [3, 4] for information on this work.

(Ad3) An exception can be made for nonisotropy when the variations of θ_l and T are in one direction only, while the anisotropy has a principal axis in the same direction.

(Ad4) Much work has been done on freezing and thawing in porous media, especially in soils. Again, the reader is referred to the literature [5–7]. A fundamental study of the phenomenon of ice lens formation was performed at the Laboratoire d'Aérothermique at Meudon by Vignes [8], who did the experimental work at the author's laboratory. Further work on this subject at our laboratory is described in ref. [9].

(Ad5) The surface phenomena are of particular importance in colloidal materials and in the study of rheological properties, see for instance refs. [10, 11]. Knudsen effects were discussed by Luikov *et al.* [12].

The following simplifying assumptions were introduced implicitly or explicitly in the derivation of the theory:

(a) The liquid in the porous medium has the properties of pure bulk water. This implies *inter alia* that no solutes or surfactants are present.

(b) Liquid movement is by viscous flow under the influence of capillary and adsorptive forces.

(c) Vapour movement is by diffusion in the gas-filled pores; apart from vapour these pores contain air as an inert gas.

(d) Free convection in the gas phase can be neglected.

(e) The total pressure is uniform and constant.

(f) Local thermodynamic equilibrium prevails throughout the porous medium. This applies in particular to the liquid in contact with its vapour.

(g) Heat transfer by radiation is negligible.

(h) The temperature dependence of ρ_l, c, h and L is neglected.

A basic explicit assumption that deserves to be mentioned separately is that a macroscopic approach can be applied. This means that the porous medium can be treated as a quasi-continuum where volume-averaged quantities replace the local ones. In addition, it is assumed that the usual calculus can be applied to the volume-averaged quantities. During the past decades a solid basis for this approach was provided by the works of Whitaker [13], Slattery [14], Gray [15] and others.

3. EXPERIMENTS

Comparisons have been made between the outcome of experiments and predictions according to the theory. Both laboratory experiments and field data were used. The comparison has led to conflicting evidence. In some cases a reasonable, or even good, agreement between theory and experiment was found; in others there were discrepancies, some of them large.

In interpreting these findings the following should be kept in mind:

(a) Due attention was not always paid to the theory's limitations and its underlying assumptions.

(b) For a proper test of the theory the moisture content and the temperature must be measured with a high degree of accuracy since their gradients must be known. Measurements of the $\psi(\theta_l)$ and $K(\theta_l)$ relations are not easy and are often time consuming.

(c) In field experiments one has to cope with the inevitable variability of natural soils, while unwanted material in solution may also complicate the experiments. In most cases, the influence of hysteresis will not be negligible. Furthermore, soil samples must be taken for determining various soil properties in the laboratory.

In the following, the outcome of some of the experimental work is discussed briefly. Here a somewhat arbitrary selection has been made as well.

3.1. *Laboratory experiments*

In this subsection the doctoral dissertations of van der Kooi [16], Jury [17] and Crausse [18] are discussed first.

Heat and moisture transfer in cellular concrete roofs was studied by van der Kooi. Cellular concrete is not a granular material; it has two types of pores, i.e. relatively large ones (the cells) and micropores within the concrete. He found that the behaviour of slabs of cellular concrete could be described by the theory, although some adjustment of the quantities D_θ and D_T (see Appendix, equation (A9)) was needed to obtain close agreement between theory and experiment.

Jury studied heat and moisture transfer in a sand. The agreement between theory and experiment was only partial. Jury also showed how the flux equations based on the theory of thermodynamics of irreversible processes [19] are related to the 'mechanistic ' equations of Philip and de Vries.

Crausse did a comprehensive study of heat and moisture transfer in a sand. He found reasonable to good agreement between calculated and measured values of the macroscopic diffusivities, with one exception: his measured value of $D_{\theta v}$ was nearly an order of magnitude larger than the theoretical one. No explanation could be offered. Crausse also found close agreement between the measured temperature dependence of ψ and K and that predicted on the basis of the change with temperature of the surface tension and the kinematic viscosity of water.

Koopmans [20] listed a number of cases where large discrepancies between experiment and theory were found. These refer *inter alia* to an anomalous temperature dependence of ψ and K, and to experimental D_{Tl}-values that were much larger than the calculated ones (also found by Jury).

In The Netherlands recently an extensive study was made of heat and moisture transfer in sandy soils around power cables by Groeneveld *et al.* [21]. Apart from laboratory studies, some field experiments were done [22]. The theory was applied, using the λ^*-representation for $\mathbf{q}_h$ (see equations (A10) and (A11)). Here also some adjustment proved to be necessary to obtain agreement between theory and experiment.

3.2. *Field experiments*

Much field work was performed by Jackson *et al.* [23]. They found good agreement between theory and experiment at night, but some large discrepancies during the day, especially in the uppermost soil layers. A similar observation was made by Wierenga and de Wit [24] in a study of heat transfer in a natural soil.

In Australia, Rose [25] investigated moisture transport in a field soil during six consecutive days with an amplitude of the diurnal temperature cycle at the surface of about 20°C. The terms of the water balance inside the upper soil layers were obtained from the observations by applying the theory, with the exception of the contribution of vapour flow under the influence of a temperature gradient, which was determined from the balance itself. During certain periods a much higher value for this term was found than the theoretical one. In interpreting this result it should be noted that all experimental and theoretical shortcomings will appear in this remaining term, as was recognized by Rose.

In concluding this section it can be said that the theory has proved its usefulness in describing and analysing laboratory and field experiments. However, doubts remain about the possibility of making sufficiently accurate predictions on the basis of the theory. These doubts arise not only from the limitations of the theory, but also from possible shortcomings of the experimental procedures and inaccuracies of the experimental results.

4. HYSTERESIS

Hysteresis in the $\psi(\theta_l)$ relation occurs because of the particular geometry of the pore space, consisting of caverns connected by narrow passages or necks, as was already observed in 1930 by Haines [26]. The reader is referred to Childs [27], in particular Sections 8.5, 8.6, 10.7 and 11.2, for a discussion of this phenomenon and its description by the so-called independent domain model.

As a consequence of hysteresis, ψ becomes a multivalued function of θ_l, its actual value depending on the history of changes of moisture content. Equation (A1) with the expression (A2) for $D_{\theta l}$ can still be applied as long as, starting from a given moisture distribution, a monotonic increase or decrease of θ_l occurs. When a reversal of the trend sets in, an extra term arises in equation (A1). According to Childs the expression for q_{lx} becomes

$$q_{lx} = -\rho_l\left(D_{\theta l}\frac{\partial\theta_l}{\partial x} + K\frac{\partial\psi}{\partial\theta_{lr}}\frac{\partial\theta_{lr}}{\partial x}\right) \tag{1}$$

where θ_{lr} is the moisture content at the onset of reversal. Corresponding expressions hold for q_{ly} and q_{lz}. Childs demonstrates that the terms in brackets are of opposite sign and can be of the same order of magnitude. Hence, hysteresis tends to reduce the differences in ψ and thereby the value of $\mathbf{q}_l$. The situation becomes even more complicated when consecutive reversals occur in the course of time. Childs does not discuss non-isothermal situations.

The hydraulic conductivity is also subject to hysteresis, since at the same moisture content the distribution of water over the pore space can be different. Fortunately, the influence is a small one and it can usually be neglected in practice. This can be understood by observing that θ_l determines the average cross-section for liquid flow, whilst the main flow resistance is located in the narrow pores and passages.

In most of the experiments mentioned in Section 3 hysteresis did occur. In the field this is due to the variable weather conditions and the diurnal and annual temperature cycles. A case where hysteresis may be neglected is that of infiltration of water in soils from natural or artificial sources. In technology, drying under proper conditions may represent such a case.

In two of these investigations the influence of hysteresis was accounted for in a purely ad hoc manner. Crausse [18] increased the value of $\partial\psi_d/\partial\theta_l$ (here subscript d refers to the boundary drying curve) by a factor 2 or 4 when rewetting took place, in studying the redistribution of moisture in a sealed sample due to the application of a temperature difference. With the factor 2 the closest agreement between experimental and theoretical values was obtained.

Groeneveld *et al.* [21] started from the boundary wetting curved, $\psi_w(\theta_l)$, and changed its slope by a factor $1/e(\theta_l) \leqslant 1$, where

$$\begin{aligned} e(\theta_l) &= 1, && \text{for } -\psi \geqslant 1\,\text{m} \\ e(\theta_l) &= 1-\log(-\psi_w^{(m)}), && \text{for } -\psi \leqslant 1\,\text{m}. \end{aligned} \tag{2}$$

Superscript (m) denotes that the numerical value of ψ_w, expressed in m, is taken. A new $\psi(\theta_l)$ curve, meant to be used in non-isothermal situations, was obtained by integration

$$\psi(\theta_l) = \int_\varepsilon^{\theta_l}\frac{\partial\psi}{\partial\theta_l'}\frac{d\theta_l'}{e(\theta_l')} + \psi(\varepsilon) \tag{3}$$

where ε is a low θ_l-value for which the hysteresis loop is closed. This procedure also led to an improved agreement between theory and experiment.

An interesting observation was made by Highgate and Parr [28]. In laboratory experiments they found a very rapid progress of the drying front in two sands when a critical temperature difference between the dry and the wet regions was exceeded. They pointed out that under certain conditions the value of the suction $-\psi_d(\theta_{ld}, T_d)$, at a certain location in the dry part of the sample, can be lower than the suction, $-\psi_w(\theta_{lw}, T_w)$, at some location in the wetter part, although $\theta_{ld} < \theta_{lw}$. They ascribed this to the increase of $-\psi$ with decreasing T. Hence, if $T_d - T_w$ is large enough the condition

$$-\psi_d(\theta_{ld}, T_d) < -\psi_w(\theta_{lw}, T_w) \tag{4}$$

might be satisfied, depending on the vertical width of the hysteresis loop in the $(\theta_l, -\psi)$ plane. In such a case, transport in the liquid and in the vapour phases would be in the same direction, provided that $p_v(\theta_{ld}, T_d) > p_v(\theta_{lw}, T_w)$. This could explain the rapid advance of the drying front. An 'anomalous' large value of $\partial\psi/\partial T$ would promote such a situation. The same could be said of a slight change of $-\psi_w$ with increasing θ_l, because of the influence of hysteresis as described in connection with equation (1), at the wet end.

In conclusion, it can be said that:

(a) At present there is no indication of the possibility of considerable progress on the theoretical side because of the extreme complexity of the phenomenon.

(b) An experimental approach aimed at studying the behaviour of non-isothermal systems subjected to hysteresis, with the purpose of developing a semi-empirical procedure for taking hysteresis into account, shows some promise.

5. THERMAL CONDUCTIVITY

In ref. [2] two representations of the equation for the heat flux density were proposed (equations (A10) and (A11)). They differ in the value assigned to the apparent thermal conductivity, here denoted by λ^* and λ^+.

Equation (A10) follows from the expression

$$\mathbf{q}_h = -\lambda'\nabla T + H_l\mathbf{q}_l + H_v\mathbf{q}_v \tag{5}$$

where H is the specific enthalpy. The first term represents pure conduction (λ' was denoted by λ_* in ref. [2]), the second and third terms enthalpy transport by liquid and vapour flow, respectively. It is assumed that macroscopic heat transfer by conduction and by convection are additive. With

$$H_l = c_l(T-T_0) \tag{6}$$

$$H_v = L(T_0)+c_{pv}(T-T_0) = L(T)+c_l(T-T_0) \tag{7}$$

we find

$$\mathbf{q}_h = \lambda'\nabla T + c_l(T-T_0)\mathbf{q}_m + L\mathbf{q}_v. \tag{8}$$

Equation (A10) then follows immediately from equations (8) and (A3), with

$$\lambda^* = \lambda' + L\rho_l D_{Tv} \tag{9}$$

as an apparent thermal conductivity which incorporates transfer of latent heat by vapour movement under the influence of a temperature gradient.

An alternative way of incorporating this phenomenon goes back to Krischer and Rohnalter [29] and was extended in ref. [1] to cases where $h < 1$. In this approach, a 'microscopic' apparent thermal conductivity of the gas phase is introduced. We denote this quantity by λ_a^+; as will be shown below, it is given by

$$\lambda_a^+ = \lambda_a + \lambda_v = \lambda_a + h\lambda_{vs}$$

$$= \lambda_a + LD\frac{P}{P-p_v}\frac{hM}{RT}\frac{\mathrm{d}p_{vs}}{\mathrm{d}T} \tag{10}$$

λ_{vs} increases rapidly with temperature; at 0°C it is approximately equal to λ_a, at 60°C to λ_l (liquid water). The value of the diffusion coefficient of water vapour in air can, according to ref. [30], be obtained from†

$$D = D_0(p_0/p)(T/T_0)^{1.88} \tag{11}$$

with $p_0 = 101325$ Pa, $T_0 = 273.16$ K and $D_0 = 21.7\times10^{-6}\ \mathrm{m^2\,s^{-1}}$.

The macroscopic apparent thermal conductivity following from the substitution of λ_a by λ_a^+ is denoted by λ^+, where $\lambda^+ \geqslant \lambda^*$. The advantage of this procedure is that it locates the contribution of latent heat transfer by vapour movement due to ∇T inside the gas filled pores, where it is often dominant, instead of adding its volume-averaged value to the other forms of heat transfer, as is done in equation (5).

The difficulties that confront us here are due to the fact that heat transfer by conduction and by convection are not additive but are intertwined in a complicated way on the microscale. On this scale the additivity of λ_a and λ_v in equation (10) is also questionable. The flow path of heat conduction will usually not be the same as that of latent heat transfer by vapour movement. This is particularly so when series–parallel transfer of moisture by vapour distillation and capillary transfer through liquid islands, as proposed in ref. [1], is operative. However, only a complete microscopic treatment could take these effects into account. The procedure adopted here is supported by the outcome of λ-measurements.

In the λ^+-representation an asymmetry is introduced with respect to latent heat transfer by vapour movement under the influence of a gradient of moisture content. This part remains represented by the last term on the right-hand side of equation (A11). However, in many cases of practical interest this term is quite small compared to $-\lambda^+\nabla T$.

An expression for the vapour flux density in the gas phase can be easily derived. We shall denote it by $\mathbf{q}_v^\mu$, where the superscript indicates that it is a microscopic quantity. Note that it is not a local quantity but a phase average in the terminology of Whitaker [13].

The basic expression for $\mathbf{q}_v^\mu$ is

$$\mathbf{q}_v^\mu = -D\frac{P}{P-p_v}\frac{M}{RT}\nabla p_v. \tag{12}$$

Using equations (A6), (A8) and neglecting the term with $\partial h/\partial T$, we obtain

$$\nabla p_v = hp_{vs}\frac{MgD_{\theta l}}{RTK}\nabla\theta_l + \zeta h\frac{\mathrm{d}p_{vs}}{\mathrm{d}T}\nabla T. \tag{13}$$

From Clapeyron's equation we have

$$\mathrm{d}p_{vs}/\mathrm{d}T = LMp_{vs}/RT^2 \tag{14}$$

so that

$$\mathbf{q}_v^\mu = -D\frac{P}{P-p_v}\frac{M\rho_v}{RT}\left(\frac{D_{\theta l}g}{K}\nabla\theta_l + \frac{\zeta L}{T}\nabla T\right). \tag{15}$$

From this equation it can be determined whether or not in a given case the term with $\nabla\theta_l$ is negligible compared to that with ∇T. If it is, the λ^+-representation can be applied, neglecting the last term on the right-hand side of equation (A11).

Experimental values of λ^+ can be obtained by using a non-stationary method with little redistribution of moisture during the time of measurement. When starting from a uniform moisture content, this implies that $\nabla\theta_l$ will be small throughout the sample. An analysis

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† The original values of Krischer and Rohnalter [29] were too high.

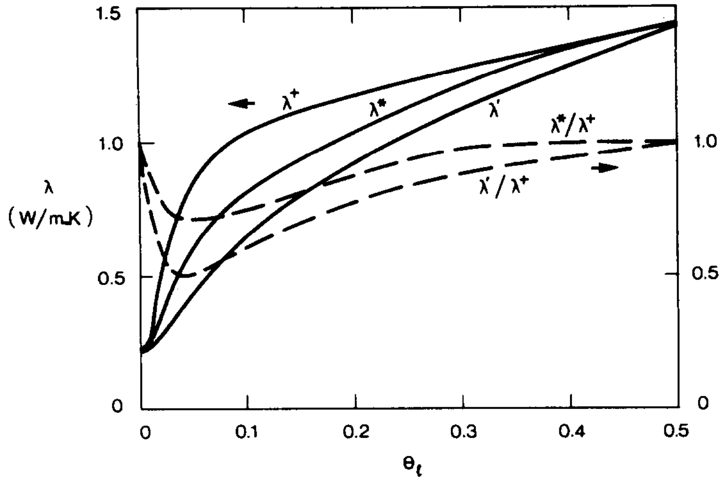

FIG. 1. Comparison of λ', λ^* and λ^+ for light clay at 40°C. The broken curves give the ratios λ'/λ^+ and λ^*/λ^+.

of the non-stationary cylindrical probe method was presented in ref. [31].

In ref. [2], I have already treated the case of steady-state heat conduction for a closed system ($\mathbf{q}_m = 0$). The movement of moisture due to a temperature gradient is then compensated by a return flow under the influence of $\nabla\theta_l$. At the dry end of the moisture range, where $D_{\theta l} \ll D_{\theta v}$, there will be no net vapour movement and $\mathbf{q}_l$ and $\mathbf{q}_v$ will both be zero. So there is no convective heat transfer and we are left with

$$\mathbf{q}_h = -\lambda'\nabla T. \tag{16}$$

Note that the λ^*-representation will give the same result.

In ref. [2] I suggested a modification of the expression for λ_v (see equation (10)) by addition of a factor $D_{\theta l}/D_\theta$, which represents the ratio of the liquid return flow to the total flow. This suggestion applies only to steady states with $\mathbf{q}_m = 0$.

It should be stressed that in calculating λ from experimental data the proper equation for $\mathbf{q}_h$ should be used, and not simply $\mathbf{q}_h = -\lambda\nabla T$.

Calculated values of λ', λ^* and λ^+ can be obtained by application of a method I developed for predicting the thermal conductivity of soils from their composition, porosity, particle shape and moisture content [32]. In order to obtain an insight in the differences between these three quantities, I calculated them for a silica sand and a light clay soil, for temperatures of 20, 40 and 60°C. The results are published elsewhere [33]. A typical example is given in Fig. 1. It holds for the clay soil ($\lambda_M = 2.9$ W m^{-1} K^{-1}, $S = 0.5$, shape factor of oblate spheroids $= 0.125$) at a temperature of 40°C. The three conductivities are the same for $\theta_l = 0$ and S, because there is no vapour movement in these cases. The largest differences occur at low moisture contents.

Unfortunately no general rule can be given for which of the two representations should be used. I would recommend use of the λ^+-form generally as a first choice, checking its applicability from equation (15) in case of doubt. Particular care should be exercised in the region where $D_{\theta l} \leqslant D_{\theta v}$, and proper judgement should be applied by considering the prevailing moisture conditions.

What was said before about the thermal conductivity applies primarily to granular porous media. If, for instance, we have a porous medium with a continuous solid matrix, with a thermal conductivity much larger than that of water, the influence of the vapour transfer will be less than for a granular medium with the same porosity and consisting of particles having the same λ_M.

In an analysis of a practical situation it is necessary to solve the coupled partial differential equations (A12) and (A13) numerically. Developments during the past decades of computers have greatly extended the possibilities of handling these complicated non-linear equations.

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APPENDIX

Mathematical formulation of the theory of Philip and de Vries

Liquid flux density

$$\mathbf{q}_\mathrm{l} = -\rho_\mathrm{l}(D_{\theta\mathrm{l}}\nabla\theta_\mathrm{l} + D_{T\mathrm{l}}\nabla T + K\mathbf{k}) \qquad \text{(A1)}$$

$$D_{\theta\mathrm{l}} = K\partial\psi/\partial\theta_\mathrm{l}, \quad D_{T\mathrm{l}} = K\partial\psi/\partial T. \qquad \text{(A2)}$$

Vapour flux density

$$\mathbf{q}_\mathrm{v} = -\rho_\mathrm{l}(D_{\theta\mathrm{v}}\nabla\theta_\mathrm{l} + D_{T\mathrm{v}}\nabla T) \qquad \text{(A3)}$$

$$D_{\theta\mathrm{v}} = f(a)D\frac{P}{P-p_\mathrm{v}}\frac{Mg}{RT}\frac{\rho_\mathrm{v}}{\rho_\mathrm{l}}\frac{\partial\psi}{\partial\theta_\mathrm{l}} \qquad \text{(A4)}$$

$$D_{T\mathrm{v}} = f(a)D\frac{P}{P-p_\mathrm{v}}\frac{\rho_\mathrm{v}}{\rho_\mathrm{l}}\frac{\zeta}{p_\mathrm{vs}}\frac{\mathrm{d}p_\mathrm{vs}}{\mathrm{d}T} \qquad \text{(A5)}$$

$$\mathbf{p}_\mathrm{v} = hp_\mathrm{vs} = p_\mathrm{vs}\exp(Mg\psi/RT) \qquad \text{(A6)}$$

$$f(a) = a+\theta_\mathrm{l} = S, \qquad \text{for } \theta_\mathrm{l} \leqslant \theta_\mathrm{lk} \qquad \text{(A7)}$$

$$f(a) = a + a(S-a)/(S-\theta_\mathrm{lk}), \quad \text{for } \theta_\mathrm{l} > \theta_\mathrm{lk}$$

$$\zeta = (\nabla T)_\mathrm{a}/\nabla T. \qquad \text{(A8)}$$

Total moisture flux density

$$\mathbf{q}_\mathrm{m} = \mathbf{q}_\mathrm{l} + \mathbf{q}_\mathrm{v} = -\rho_\mathrm{l}(D_\theta\nabla\theta_\mathrm{l} + D_T\nabla T + K\mathbf{k}). \qquad \text{(A9)}$$

Heat flux density

$$\mathbf{q}_\mathrm{h} = -\lambda^*\nabla T + c_\mathrm{l}(T-T_0)\mathbf{q}_\mathrm{m} - L\rho_\mathrm{l}D_{\theta\mathrm{v}}\nabla\theta_\mathrm{l} \qquad \text{(A10)}$$

or

$$\mathbf{q}_\mathrm{h} = -\lambda^+\nabla T + c_\mathrm{l}(T-T_0)\mathbf{q}_\mathrm{m} - L\rho_\mathrm{l}D_{\theta\mathrm{v}}\nabla\theta_\mathrm{l}. \qquad \text{(A11)}$$

Coupled partial differential equations for θ_l and T

$$\left\{1 + \frac{(S-\theta_\mathrm{l})\rho_\mathrm{v}}{\rho_\mathrm{l}}\frac{Mg}{RT}\frac{\partial\psi}{\partial\theta} - \frac{\rho_\mathrm{v}}{\rho_\mathrm{l}}\right\}\frac{\partial\theta_\mathrm{l}}{\partial t} + \frac{(S-\theta_\mathrm{l})h}{\rho_\mathrm{l}}\frac{\mathrm{d}\rho_\mathrm{vs}}{\mathrm{d}T}\frac{\partial T}{\partial t}$$

$$= \nabla(D_\theta\nabla\theta_\mathrm{l}) + \nabla(D_T\nabla T) + \partial K/\partial z \qquad \text{(A12)}$$

$$\left\{\frac{L(S-\theta_\mathrm{l})\rho_\mathrm{v}Mg}{RT}\frac{\partial\psi}{\partial\theta_\mathrm{l}} - L\rho_\mathrm{v}\right\}\frac{\partial\theta_\mathrm{l}}{\partial t}$$

$$+ \left\{C + L(S-\theta_\mathrm{l})h\frac{\mathrm{d}\rho_\mathrm{vs}}{\mathrm{d}T}\right\}\frac{\partial T}{\partial t} = \nabla(\lambda^*\nabla T)$$

$$+ L\rho_\mathrm{l}\nabla(D_{\theta\mathrm{v}}\nabla\theta_\mathrm{l}) - c_\mathrm{l}(\mathbf{q}_\mathrm{l}\nabla T) - c_{p\mathrm{v}}(\mathbf{q}_\mathrm{v}\nabla T). \qquad \text{(A13)}$$

In the alternative formulation λ^+ must be substituted for λ^*

$$C = \rho_\mathrm{M}c_\mathrm{M}(1-S) + \rho_\mathrm{l}c_\mathrm{l}\theta_\mathrm{l} + \rho_\mathrm{v}c_{p\mathrm{v}}a. \qquad \text{(A14)}$$

REEXAMEN DE LA THEORIE DU TRANSFERT DE CHALEUR ET D'HUMIDITE DANS LES MILIEUX POREUX

Résumé—On présente l'état actuel de la théorie du transfert simultané de chaleur et d'humidité dans les milieux poreux développée par J. R. Philip et l'auteur dans les années 1950. Tout d'abord, on porte l'attention sur les limites de la théorie et sur les hypothèses qu'elle contient. Ensuite, on discute des vérifications expérimentales de cette théorie, pour conclure que celle-ci est utile pour décrire et analyser les expériences, mais que des doutes subsistent quant aux valeurs calculées. Ces doutes sont la conséquence: (a) des limites de la théorie; (b) de l'incertitude sur la qualité des procédures expérimentales et des données. Des remarques sont faites sur l'hysteresis et son influence possible. On conclut que des expériences sont nécessaires sur le comportement des systèmes non isothermes soumis à l'hysteresis. Finalement le problème de la définition et de l'utilisation d'une conductivité thermique apparente est analysé. Dans les articles originaux était présentée une alternative. On dérive une expression pour la moyenne de phase de la densité de flux de vapeur. Un exemple numérique est présenté et des suggestions sont faites concernant le choix convenable dans l'alternative.

THEORIE DES WÄRME- UND FEUCHTETRANSPORTS IN PORÖSEN MEDIEN

Zusammenfassung—Diese Arbeit gibt einen Überblick über den gegenwärtigen Kenntnisstand in der Theorie des gleichzeitigen Wärme- und Feuchtetransports in porösen Medien, wie sie von J. R. Philip und dem Autor Mitte der 1950er Jahre erarbeitet worden ist. Zuerst werden die Einschränkungen der Theorie und die Annahmen betrachtet. Als nächstes werden Labor- und Feldversuche zur Überprüfung der Theorie kurz diskutiert, was zu dem Ergebnis führt, daß der Nutzen der Theorie bei der Beschreibung und Analyse der Versuche bestätigt wurde, jedoch Zweifel an ihrem Wert bei der Vorhersage bestehen blieben. Diese Zweifel sind die Folge von (a) den Einschränkungen in der Theorie, (b) der Unsicherheit über die Güte der Experimentierverfahren und -daten. Es werden Anmerkungen zur Hysterese und ihren möglichen Einfluß gemacht. Weitere Experimente zur Untersuchung des Verhaltens nicht-isothermer Systeme, die einer Hysterese unterworfen sind, werden als notwendig betrachtet. Schließlich wird das Problem untersucht, wie eine geeignete scheinbare Wärmeleitfähigkeit definiert und angewendet werden kann. In der Originalarbeit wurden zwei Alternativen präsentiert. Ein numerisches Beispiel wird vorgestellt, Vorschläge zur geeigneten Auswahl zwischen den beiden Alternativen werden gemacht.

УТОЧНЕННАЯ ТЕОРИЯ ПЕРЕНОСА ТЕПЛА И ВЛАГИ В ПОРИСТЫХ СРЕДАХ

Аннотация—Представлено современное состояние теории связанного тепло-и влагопереноса в пористых средах, разработанной Дж. Р. Филипом и автором настоящей статьи в 50-е годы. Обсуждаются ограниченность теории, допущения, лежащие в ее основе, попытки ее подтверждения в лабораторных и натурных экспериментах; сделан вывод о возможности ее применения при описании и анализе экспериментов, однако использование при проведении расчетов ставится под сомнение. Эти сомнения являются следствием (а) ограниченности теории, (б) неопределенности оценки достоверности экспериментов и их результатов. Отмечается возможная роль гистерезиса. Сделан вывод о необходимости экспериментального изучения поведения неизотермических систем, обладающих гистерезисом. В заключение анализируется задача определения и использования истинной теплопроводности. Представлены два возможных подхода к ее решению. Выведено выражение для усредненной плотности потока пара. Дан численный пример и предложен выбор подхода к решению задачи.